# Exercise 58

(a) Prove that the equation has at least one real root. (b) Use your calculator to find an interval of length 0.01 that contains a root.

$$\ln x = 3 - 2x$$

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### Solution

Bring both terms to the same side.

$$\ln x + 2x - 3 = 0$$

The function $f(x) = \ln x + 2x - 3$ is continuous on $(0, \infty)$ because it's the sum of two functions known to be continuous on $(0, \infty)$ and $(-\infty, \infty)$, respectively, a logarithmic function and a polynomial function.

$$f(x) = 0$$

Find a value of $x$ for which the function is negative, and find a value of $x$ for which the function is positive.

$$f(1) = -1$$

$$f(2) \approx 1.69$$

$f(x)$ is continuous on the closed interval $[1, 2]$, and $N = 0$ lies between $f(1)$ and $f(2)$. By the Intermediate Value Theorem, then, there exists a root within $1 < x < 2$. Find other values of $x$ within this interval for which the function is negative and positive.

$$f(1.3) \approx -0.138$$

$$f(1.4) \approx 0.136$$

$f(x)$ is continuous on the closed interval $[1.3, 1.4]$, and $N = 0$ lies between $f(1.3)$ and $f(1.4)$. By the Intermediate Value Theorem, then, there exists a root within $1.3 < x < 1.4$. Find other values of $x$ within this interval for which the function is negative and positive.

$$f(1.34) \approx -0.0273$$

$$f(1.35) \approx 0.000105$$

$f(x)$ is continuous on the closed interval $[1.34, 1.35]$, and $N = 0$ lies between $f(1.34)$ and $f(1.35)$. By the Intermediate Value Theorem, then, there exists a root within $1.34 < x < 1.35$.